SURNAME .... FIRST NAME ....
(Block capitals, please)
JUNIOR SCHOOL .... SENIOR SCHOOL ....
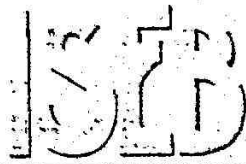
Independent Schools Examinations Board

COMMON ENTRANCE EXAMINATION AT 13+

MATHEMATICS

PAPER 4

Calculator Paper

Wednesday 26 February 2003

Please read this information before the examination starts.

- This examination is 60 minutes long.

- **All** questions should be attempted.

- A row of dots .... denotes a space for your answer.

- Where answers are not exact they should be given to three significant figures, unless specified otherwise.

- The π button on the calculator should be used for calculations involving π.

1. (i) Writing down all the figures shown on your calculator, evaluate

$$\frac{28.7 \times 1.59}{15.6 - 14.3}$$

Answer: _35.10230769_ (2)

   (ii) (a) Write your answer to part (i) correct to 1 decimal place.

Answer: _35.1_ (1)
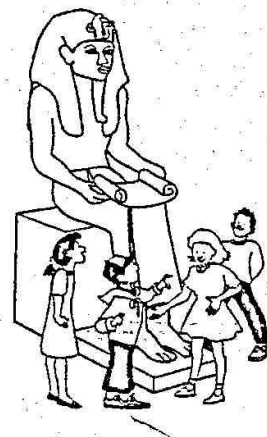
   (b) Write your answer to part (i) correct to 1 significant figure.

Answer: _40_ (1)

2. Year 8 pupils are going on a school trip to an Egyptian exhibition. There are 4 classes in Year 8. Each class has 15 children. 9 adults are also going on the trip.

   (i) How many people altogether are going on the trip?

$15(4)+9 =$

Answer: _69_ (1)

Everyone travels by minibus.

Each minibus holds 12 passengers and costs £35 to hire, including the driver.

(ii) Calculate the total cost of hiring the minibuses.

$\frac{69}{12} = 5.75$

$35(6) = 210$

Answer: £ _210_ (2)

Tickets for the exhibition are priced as follows:

children   £1.60 each
adults   £2.65 each
one adult free for every ten children

(iii) What is the total cost of the tickets for the exhibition?

60   Children, 9 adults

60   children, 6 adults free

60 (1.60) + 3(2.65) = £103.95

Answer: £ 103.95 .... (3)

(iv) If everybody on the trip pays the same amount, what is the cost per person for the whole trip?

$$\frac{210 + 103.95}{69} = 4.55$$

Answer: £ 4.55 .... (2)

3. As part of his training, Jim goes running and sprinting.

First he runs at 5 m/s for 2 minutes and 40 seconds.

(i) How far does he run?

$S \times T = D$

5×160

Answer: 800 .... m   (2)

Then he sprints 300 m at $7\frac{1}{2}$ m/s.

(ii) For how many seconds does he sprint?

$T = \dfrac{D}{S} = \dfrac{300}{7.5} = 40$

Answer: ...   40 .... s   (1)

(iii) What is his average speed for the two activities?

$AS = \dfrac{TD}{TT} = \dfrac{800 + 300}{160 + 40} = \dfrac{1100}{200} = 5.5$

Answer: 5.5 .... m/s   (2)

4. (a) Simplify the following expressions:

(i) $6p^2 + 4pq - p^2 - 7pq$

Answer: $5p^2 - 3pq$ .... (2)

(ii) $m \times 2m \times 3m \times 4m$

Answer: $24 m^4$ .... (2)

(b) Multiply out the brackets and simplify

$3m(6m - 2n) - 7n(n - 4m)$

$18m^2 - 6mn - 7n^2 + 28mn$

$\parallel$

Answer: $18m^2 + 22mn - 7n^2$ (3)

(c) Factorise completely
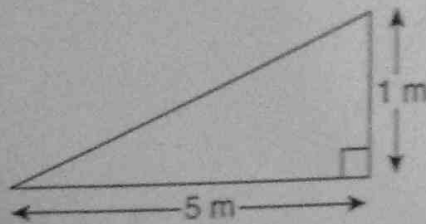
$6u^2v + 9u^3v$

Answer: $3u^2v(2 + 3u)$ (2)

5. A slope which rises 1 metre for every 5 metres travelled horizontally is said to have a gradient of 1 in 5

This can also be written as a gradient of 20% because $\frac{1}{5} = 20\%$

(i) Write down the percentage equivalents of the following slopes:

(a) 1 in 4

$\frac{1}{4} \times 25\%$
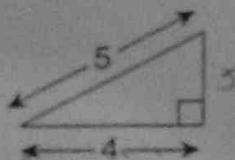
Answer: 25 _____ % (1)

(b) 3 in 8

$\frac{3}{8}$

Answer: 37.5 _____ % (2)

(c)

Answer: $\frac{4}{4}$ 75 _____ % (2)

(ii) For water to run safely off a roof, the gradient must be greater than 1 in 12
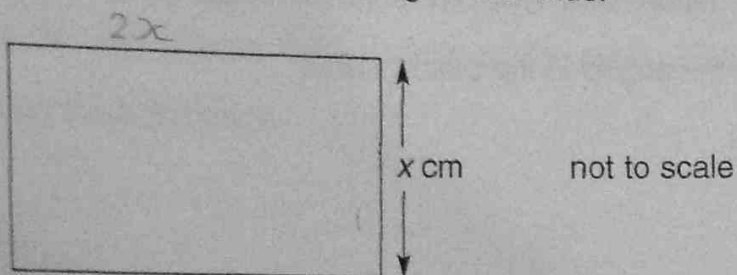
Giving a reason, state whether an 8% gradient is safe.

$\frac{1}{12} = 8.3\%$

Answer: No, $\frac{1}{12}$ = 8.3% Which is greater than

8% _____ (2)

6. (i) Here is a rectangle. It is twice as long as it is wide.

$2x$

x cm   not to scale

(a) If the width is x cm, write down an expression, in terms of x, for the perimeter of the rectangle.

$= 2x + 2x + x + x$

$= 4x + 2x$

$= 6x$

Answer: .... $6x$ .... cm (2)

(b) If the perimeter is 48 cm, what is the area of the rectangle?

$6x = 48$

$x = 8$

$8(16) = 128$

Answer: .... 128 .... cm² (2)

(ii) In another rectangle, the length is 5 cm more than the width. The perimeter of this rectangle is 70 cm.

Find the area of the rectangle.

$x+5$

x
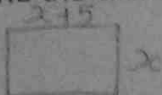
$2x + 2(x+5) = 70$

$2x + 2x + 10 = 70$

$4x = 60$

$x = 15$

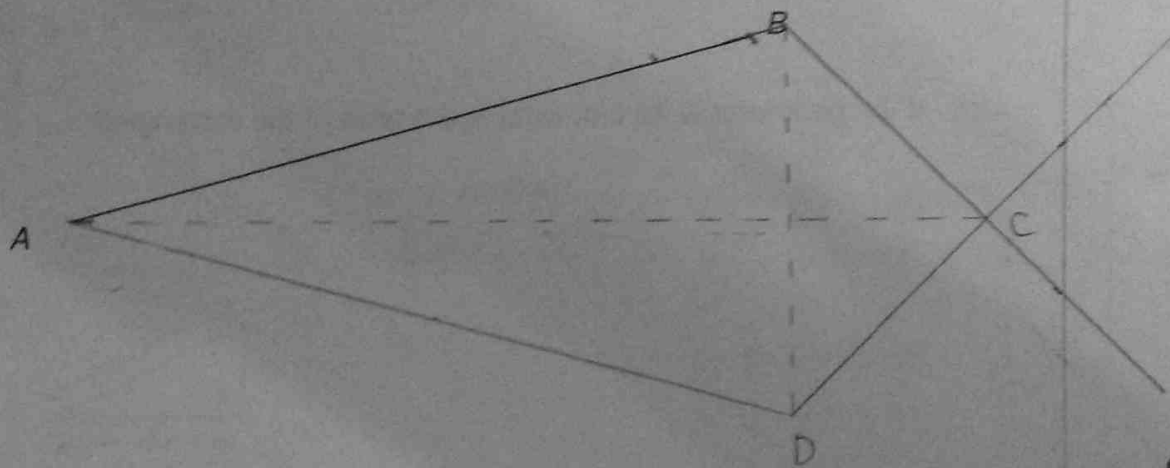Answer: .... 300 .... cm² (3)

$15(20)$

7. The angles in quadrilateral *ABCD* taken in order are in the ratio 1 : 4 : 3 : 4

   (i) Calculate the angles in the quadrilateral.

   $$\frac{360}{12} = 50°$$

   Answer: $A = 30°$ ......°, $B = 120°$ ....°, $C = 90°$ ....°, $D = 120°$ ....° (2)

   (ii) If *AB* and *AD* are each 10 cm, use the space below to make an accurate construction of quadrilateral *ABCD*. The line *AB* is drawn for you.

   (3)

   (iii) By taking suitable measurements, find the area of the quadrilateral.

   $$\frac{1}{2}(AC)(BD)$$

   $$= \frac{1}{2}(12.3)(5.3)$$

   $$= 32.595$$

   Answer: 32.6 .... cm² (3)

8. Mr Harris bought a painting for £1800

   He then sold it to an antiques shop, making a 35% profit on his buying price.

   (i) Calculate his selling price.

   Answer: £ 2430 .... (2)

   The painting was later sold by the antiques shop for £3200

   (ii) What percentage profit did the antiques shop make on the sale?
       Give your answer correct to 1 decimal place.
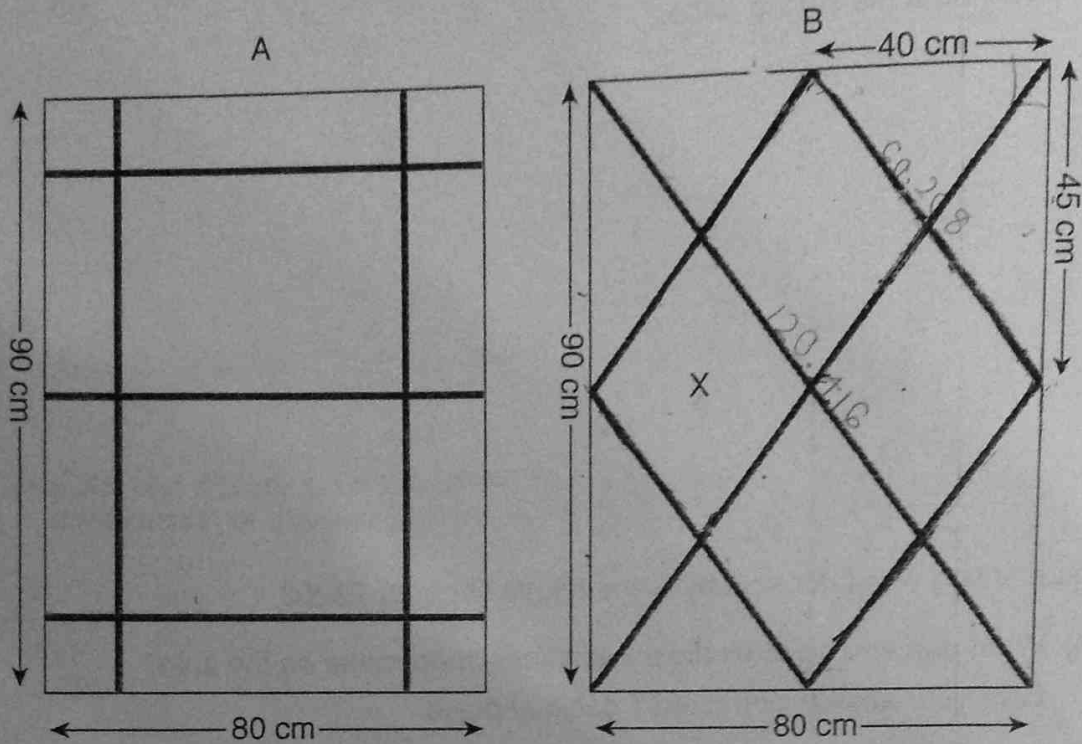
   $$\frac{3200 - 2430}{2430} \times 100$$

   Answer: 31.7 ....% (2)

   (iii) By what fraction has the painting increased in value since Mr Harris first
        bought it?

   $$\frac{3200}{1800} = \frac{32}{18} = \frac{16}{9}$$

   Answer: $\frac{16}{9}$ .... (2)

9. Two windows have decorative 'leading' as shown by the bold lines in the diagrams. Both windows are 80 cm wide and 90 cm high.

A

90 cm

80 cm

B

40 cm

45 cm

90 cm

60.208

120.416

X

80 cm

(i) Calculate the length of leading, in metres, needed for

(a) window A

$2(90) + 3(80)$
$= 420$
$= 4.20\ m$

Answer: ..4.20.. .... m   (1)

(b) window B.

$2(120.416) + 4(60.208)$
$= 481.664$
$= 4.82\ m$

Answer: ..4.82.. ...... m   (4)

(ii) Roger throws a stone at random and it hits window B.
What is the probability it hits panel X?

$\dfrac{1}{8}$

Answer: ..$\dfrac{1}{8}$.......... (2)

10. Jack and Jill are trying to find two numbers to solve a puzzle.
   - The numbers add to give 30 exactly.
   - The numbers multiply to give a number which rounds to 220 to the nearest whole number.

Jack's first suggestion is 10 and 20 because

$10 + 20 = 30$ but $10 \times 20 = 200$

Jill's first suggestion for the numbers is 14.5 and 15.5 because

$14.5 + 15.5 = 30$ but $14.5 \times 15.5 = 225$ to the nearest whole number.

Using trial and improvement, find one pair of numbers, each correct to 1 decimal place, which will solve Jack and Jill's puzzle.

$$x + y = 30$$
$$xy = 220$$

$$\Rightarrow y = 30 - x$$
$$x(30 - x) = 220$$
$$30x - x^2 = 220$$
$$x^2 - 30x + 220 = 0$$
$$x = 17.236, \ 12.764$$

$$x = 17.2, \ y = 12.8$$

$$17.2 + 12.8 = 30$$

$$17.2 (12.8) = 220.16 \Rightarrow 220$$

Answer: The numbers ....17.2........ and ....12.8........ solve the puzzle.   (6)

Turn over

11.

$$s = ut + \frac{1}{2}at^2$$

When $u = 80$ $a = 9.8$ $t = 7.5$

find

(i) ut

80(7.5)

Answer: 600 _____ ... (1)

(ii) $\frac{1}{2}at^2$

$\frac{1}{2}(80)(7.5)^2$

Answer: 275.C28 _____ ... (1)

(iii) s

Answer: 875. C25 _____ (1)

12. (i) Solve the following inequalities:

(a) $\frac{1}{3}x + 2 \leqslant 5$

$\frac{1}{3}x \leqslant 3$

$x \leqslant 9$

Answer: $x \leqslant 9$ _____ (2)

(b) $8 - 3x \leqslant 11$

$-3x \leqslant 3$

$x \geqslant -1$

Answer: ... $x \geqslant -1$ _____ (2)

(ii) List the prime numbers which satisfy both inequalities.

Answer: 2, 3, 5, 7 _____ (1)

13. The nth term of a sequence is given by

$$t_n = 3n^2 + 4$$

(i) Find the first three terms t_1, t_2 and t_3 of the sequence.

$t_n = 3n^2 + 4$

$t_1 = 3 + 4 = 7$

$t_2 = 3(4) + 4 = 10$

$t_3 = 3(9) + 4 = 31$

Answer: $t_1 = $ ..7.. ...... , $t_2 = $ ...16.......... , $t_3 = 31$ .. (3)

(ii) Find the difference between the 10th term and the 20th term.

$t_{20} = 3(400) + 4 = 1204$

$t_{10} = 3(100) + 4 = 304$

Answer: ...900............ ..... (2)

(iii) Find the smallest value of n such that $t_n > 1000$
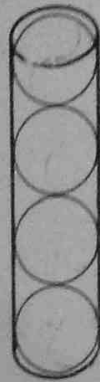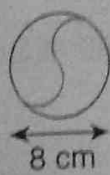
$3n^2 + 4 > 1000$

$3n^2 > 996$

$n^2 > 332$

$n > 18.2$

Answer: $n = $ ..19............................ (3)

14. A tennis ball has diameter 8 cm.

'Whizzer' tennis balls are sold in cylindrical tubes containing four balls.

8 cm

The balls just fit into the cylinder as shown in the diagram.

(i) Write down the radius of the tube.

Answer: 4 .... cm (1)

(ii) Calculate the volume of an empty cylindrical tube.

$V = \pi r^2 h = \pi (16)(32)$

$= 1608.495$

Answer: 1608.5 .... cm³ (3)

(iii) The balls take up $\frac{2}{3}$ of the volume of the tube.

What is the volume of one tennis ball?
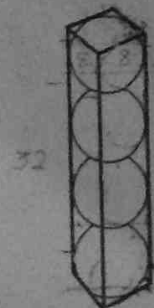
$\frac{2}{3}(1608.5) = \dfrac{1072.3}{4} = 1$ tennis ball

4 tennis balls Answer: 268.08 .... cm³ (3)

If instead the balls just fit into a square tube,

(iv) what percentage of that tube is filled by the balls?
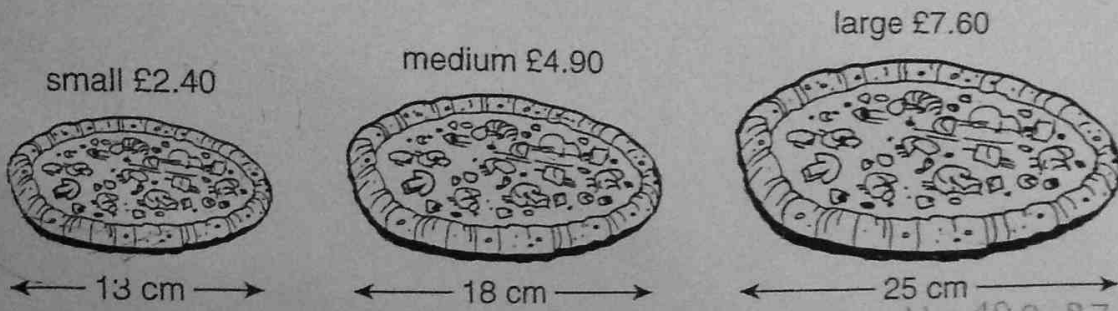
Volume of tube = 32 X 8 X 8 = 2048

32

$\dfrac{1072.3}{2048} = 0.523 \times 100$

Answer: 52% ....% (3)

S.A. 2833128

14

15. Paulo's Pizzeria has a special offer:

Buy one full price, get another the same size half price

large £7.60

small £2.40

medium £4.90

←— 13 cm —→   ←— 18 cm —→   ←———— 25 cm ————→

$\pi(7.5)^2 = 176.7 \text{ cm}^2$ $\pi(9^2) = 254.5 \text{ cm}^2$ $\pi(12.5)^2 = 490.87$

Adam wants to eat as much pizza a possible and has £8 to spend.

By considering the area of circles, investigate whether Adam should choose small, medium or large size.

Find how much change he will have from his £8

Per £:

Small $= \dfrac{176.7}{2.40} = 73.63$

Medium $= \dfrac{254.5}{4.90} = 51.94$

Large $= \dfrac{490.87}{7.60} = 64.59$

$2.40 + 1.20 + 2.40 + 1.20 = £7.20$

Answer: Adam should choose ....Small.... .... pizza.

He will have ..80.. ____ .... pence change from his £8   (8)

(Total marks: 100)